Field Solution and Propagation Characteristics of Monofilar–Bifilar Modes of Axially Slotted Coaxial Cable

ESSAM E. HASSAN

Abstract —**The electromagnetic problem of an infinite, axially slotted coaxial cable is solved. The monofilar and the bifilar eigenmodes are obtained with good accuracy and previous conflicting results are discussed. Further, a field solution satisfying the proper boundary conditions is obtained and shown to be essentially azimuthal.**

I. Introduction

Leaky axially slotted coaxial cables are used in several radio communication fields such as mine tunnel communication, intruder detection, and, quite recently, guided radar systems [1]–[5]. It is now well established that if the slotted coaxial cable is placed in free space, it supports two guided modes, both characterized as slow waves with no cutoff frequency [5]–[7]. The first one, known as the coaxial or bifilar mode, has the main part of its energy confined inside the cable with leakage outside. The second mode, known as the monofilar mode, has most of its energy confined to the outer surface of the structure with some leakage inside. Communication is established by coupling the transreceivers to the coaxial mode leakage field [7].

The fundamental problem of solving for eigenmodes and the fields inside and outside the above structure has been tackled only recently. Accurate knowledge of the propagation parameters and of the field configuration is necessary for the proper mode excitation and correct termination of this leaky structure [8]. The electrostatic solution to the problem was obtained by Kaden [9] using a conformal transformation technique. This solution, however, is restricted to air-filled cable and a vanishingly small inner conductor, which does not serve the purpose of the case considered here. In fact, the presence of the dielectric inside the cable is the key factor in limiting the effective radius of the leakage field [5]. This problem has been examined analytically by Hurd [6], who imposes some restrictions on the frequency and the cable parameters. A mode-matching technique was used by Delogne [7], but his results are conflicting. While he showed that the monofilar mode is slightly slower than the speed of light in air, a fact confirmed by the work of Hurd [6], the results for the coaxial mode show two conflicting values. In one solution he obtained a field that is faster than the velocity of light in the dielectric media, while in another the field is slower.

Manuscript received October 7, 1987; revised March 7, 1988. This work was done in the Electrical Engineering Department of the King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia.

The author is with the Department of Electrical Engineering, King Fahd University of Petroleum and Minerals, Dhahran, 31261, Saudi Arabia.

IEEE Log Number 8824243.

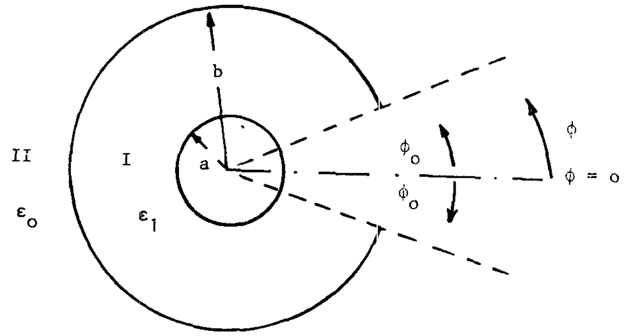

Fig. 1. Axially slotted coaxial cable.

The error in determining the proper propagation constant for the bifilar mode will considerably affect the axial field components as well as the transfer impedence [7]. It is the purpose of this paper to introduce another technique for solving the above problem and to compare the results obtained with those of [6] and [7]. The technique developed here was used earlier to solve the microstrip problem [10], [11] and is based on a stationary expression for the propagation constant of slotted waveguide developed by Rumsey [12]. The results obtained agree well with those of Hurd [6] and show that the coaxial mode is indeed faster than the TEM waves inside the cable. In addition, the technique solves for the fields inside and outside the structure. Some of the results obtained are presented. Details of the technique are to follow.

II. Formulation of the Problem

A. Field Components

Fig. 1 shows a slotted coaxial cable. The radii of the inner and outer conductors are, respectively, a and b. Region I, defined by $a \leqslant \rho \leqslant b$, is filled with a dielectric medium with permitivity ϵ_1. Region II, defined by $b < \rho$, is free space. Parameters pertaining to each region will be distinguished by the appropriate subscript or superscript.

The fields in region I or II are all hybrid and it is convenient to derive them from the z components Π_e, Π_m

of the electric and magnetic Hertz vectors. Thus for region I we have

$$\Pi_e^{(1)} = \sum_{n=0}^{\infty} A_n^{(1)} Z_n(\beta_1\rho)\cos n\phi$$

$$\Pi_m^{(1)} = \sum_{n=1}^{\infty} B_n^{(1)} \xi_n(\beta_1\rho)\sin n\phi \qquad (1)$$

where $\exp j(\omega t - \alpha z)$ is implied and

$$\beta_1 = \sqrt{K_1^2 - \alpha^2} \qquad K_1 = K_0\sqrt{\epsilon_1/\epsilon_0},$$

$$K_0^2 = \omega^2\mu_0\epsilon_0.$$

Also, $A_n^{(1)}$ and $B_n^{(1)}$ are unknown coefficients to be determined, and

$$Z_n(\beta_1\rho) = J_n(\beta_1\rho) - J_n(\beta_1 a)Y_n(\beta_1\rho)/Y_n(\beta_1 a)$$

$$\xi_n(\beta_1\rho) = J_n(\beta_1\rho) - J_n'(\beta_1 a)Y_n(\beta_1\rho)/Y_n'(\beta_1 a)$$

where the prime indicates derivative with respect to the argument of the Bessel functions.

Similarly, in region II

$$\Pi_e^{(2)} = \sum A_n^{(2)} H_n^{(2)}(\beta_2\rho)\cos n\phi$$

$$\Pi_m^{(2)} = \sum B_n^{(2)} H_n^{(2)}(\beta_2\rho)\sin n\phi$$

$$\beta_2 = \sqrt{K_0^2 - \alpha^2}. \qquad (2)$$

$A_n^{(2)}$ and $B_n^{(2)}$ are constants to be determined and $H_n^{(2)}$ are the Hankel functions of the second kind. Our aim is to find the eigenvalues α's and to determine the coefficients $A_n^{(1)}$, $B_n^{(1)}$, $A_n^{(2)}$, and $B_n^{(2)}$.

The field components everywhere may be obtained using the relations

$$E = \nabla\nabla\cdot\Pi_e + \omega^2\mu_0\epsilon\Pi_e - j\omega\mu_0\nabla\times\Pi_m$$

$$H = \nabla\nabla\cdot\Pi_m + \omega^2\mu_0\epsilon\Pi_m + j\omega\epsilon\nabla\times\Pi_e.$$

Of particular interest are the tangential electric field components $E_\phi^{(i)}$, $E_z^{(i)}$ $(i=1,2)$. Using the orthogonality relations it is simple to show that the coefficients $A_n^{(1)}$, $B_n^{(1)}$ may be determined as

$$A_n^{(1)} = \frac{j\omega\epsilon_1}{\epsilon_n\cdot\pi\beta_1^2 Z_n(\beta_1 b)} I_z(n), \qquad \begin{matrix}\epsilon_n = 1 & n\neq 0\\ = 2 & n = 0\end{matrix}$$

$$B_n^{(1)} = \frac{1}{\beta_1\pi\xi_n'(\beta_1 b)}\left[I_\phi(n) - \frac{jn\alpha}{\beta_1^2 b} I_z(n)\right] \qquad (3)$$

where

$$I_\phi(n) = \int_{-\phi_0}^{\phi_0} E_\phi|_{\rho=b}\sin n\phi\, d\phi$$

$$I_z(n) = \int_{-\phi_0}^{\phi_0} E_z|_{\rho=b}\cos n\phi\, d\phi. \qquad (4)$$

Similar expressions for $A_n^{(2)}$ and $B_n^{(2)}$ with appropriate Bessel functions and parameters may be obtained. The field components everywhere are now all determined once the tangential electric field along the slot is known.

B. Stationary Expression of α

It has been shown by Rumsey [12] and by Harrington [14] that a stationary expression for the propagation constant of a circular cylinder axially slotted waveguide may be written as

$$\int_{-\phi_0}^{\phi_0}\left[E_\phi\left(H_z^{(1)} - H_z^{(2)}\right) + E_z\left(H_\phi^{(1)} - H_\phi^{(2)}\right)\right]\Big|_{\rho=b} d\phi = 0. \qquad (5)$$

The stationary nature of the above expression with respect to the propagation constant α is further analyzed by Walter [13] and by Harrington [14] in his treatment of the propagation along a slotted cylinder. Turning back to our problem and substituting the field components obtained in subsection A, which are in terms of E_ϕ and E_z, into the above stationary expression one obtains, after some mathematical manipulations,

$$\sum_{n=1}^{\infty} I_\phi^2(n)U(n) - \sum_{n=1}^{\infty} nI_\phi(n)I_z(n)S(n) + \sum_{n=0}^{\infty} I_z^2(n)T(n) = 0 \qquad (6)$$

where

$$U(n) = \beta_1^2\frac{b}{n}Q(n) + \beta_2^2\frac{b}{n}, \qquad n = 1,2,\cdots,\infty$$

$$S(n) = 2j\alpha[Q(n)+1], \qquad n = 1,2,\cdots,\infty$$

$$T(n) = \frac{n}{b}[Q(n)+1], \qquad n = 1,2,\cdots,\infty$$

$$T(0) = -\frac{K_0^2}{2}\left(\frac{\epsilon_1/\epsilon_0}{\beta_1^2 b\ln(a/b)} - \frac{1}{\beta_2^2 b\ln(j\beta_2 b)}\right), \qquad n = 0$$

with

$$Q(n) = \left((b/a)^{2n}+1\right)\Big/\left((b/a)^{2n}-1\right).$$

In the development of (6), the $T(0)$ value is obtained using the limiting value of Bessel functions for small argument. Notice that (6) is an identity and no approximation is yet introduced.

C. Solution of the Problem

Recall that (6) is a stationary expression for α. Therefore any reasonable representation of E_ϕ and E_z on the slot must lead to a very good approximation to α, especially if one assumes the fields to be in the form of an expansion of reasonable functions with unknown coefficients, i.e.,

$$E_\phi = \sum a_j f_j(\phi) \qquad E_z = \sum b_k\psi_k(\phi). \qquad (7)$$

Coefficients a_j and b_k can be optimized using the Ritz method, which leads to the solution of the field in addition to the desired propagation constant α. The success of this method depends on the choice of the functions $f_j(\phi)$ and

$\psi_k(\phi)$. We have attempted four different sets as follows:

$$E_\phi = \sum_J a_J \sin\left(\frac{J\pi\phi}{\phi_0}\right) \Big/ \sqrt{1-(\phi/\phi_0)^2}$$
$$E_z = \sum_k b_k \cos\left(\frac{k\pi\phi}{\phi_0}\right) \cdot \sqrt{1-(\phi/\phi_0)^2} \tag{8}$$

$$E_\phi = \sum a_J \sin\left(\frac{J\pi\phi}{\phi_0}\right) \Big/ \left(1-(\phi/\phi_0)^2\right)^{3/2}$$
$$E_z = \sum_k b_k \cos\left(\frac{k\pi\phi}{\phi_0}\right) \cdot \sqrt{1-(\phi/\phi_0)^2} \tag{9}$$

$$E_\phi = \sum a_J \sin\left(\frac{J\pi\phi}{\phi_0}\right)$$
$$E_z = \sum_k b_k \cos\left(\frac{k\pi\phi}{\phi_0}\right) \tag{10}$$

$$E_\phi = \frac{a_0 \sin\phi}{\sqrt{1-(\phi/\phi)^2}} + \sum_{J=1} a_J \sin\left(\frac{J\pi\phi}{\phi_0}\right)$$
$$E_z = \sum_k b_k \cos\left(\frac{k\pi\phi}{\phi_0}\right). \tag{11}$$

The first set (eqs. (8)) is the natural assumption which is used by Delogne [7]. However it does not satisfy the boundary conditions that E_ϕ be singular as $\phi^{-1/2}$ as it approaches the edge. The set (9), although it satisfies this boundary condition, is not in the well-known form of the static field at the slot. When set (8) or (9) is substituted into (6) and (7), the coefficients a_J and b_k of (7) are highly unstable, producing highly oscillating values of (6) around the zero. This choice is then abandoned in favor of sets (10) and (11). The assumption (11) satisfies the boundary conditions. Moreover, if the singular term in (11) is taken to the left-hand side, then this side is presumably nonsingular and is represented by the set $\Sigma a_j \sin(j\pi\phi/\phi_0)$. The singularity which is assumed by one term in (11) agrees with the findings of Hurd [6] for the same problem. Notice also that we did not force the field E_z (in the representations given by (10) and (11)) to approach zero at the edge. This is not necessary since the complete set expansion is capable of representing the boundary condition at this point. Further, one would gain extra confidence in the technique once the obtained solution approached zero at the edge without such enforcement. This result is presented below, in Section III.

Assumptions (10) and (11) give highly stable and convergent solutions for α that agree well with each other and with those of Hurd [6] and Delogne [15]. They also agree with one set of solutions presented by Delogne and Laloux [7] and show that the other solution presented in [7] may not be correct. This is shown in some detail in the next section.

III. Numerical Results

For the sake of comparison with published data, we present numerical results for the following parameters: $a = 4.375$ mm, $b = 1.055$ mm, $\epsilon_1 = 1.465\epsilon_0$, and $\phi_0 = 45°$ at frequencies of 250 and 500 MHz. In addition, some results for $\phi = 20°$ are also introduced. Table I summarizes some of the results in comparison with other available data. For convenience the table presents the quantities β_m/K_0 and $\beta_c/(K_0\sqrt{\epsilon_1/\epsilon_0})$, where β_m and β_c are, respectively, the monofilar mode and coaxial mode propagation constants. Convergence of the above results for the two sets of fields (10) and (11) differs considerably. For set (11), convergence is very fast and quite stable. In fact, with the number of slot field expansion as low as (2,2), the results are as exact as those of an expansion of (8,8) terms. This is best shown in Table II.

TABLE I
PROPAGATION CONSTANTS $\beta_1 = \beta_m/K_0$ AND $\beta_2 = \beta_c/(K_0\sqrt{\epsilon_1/\epsilon_0})$ COMPARED WITH OTHER DATA

		Set (10)	Set (11)	Ref. [6]	Ref. [7]
β_1	$F = 250$	1.00132	1.00108	1.0010	1.0014
	$F = 500$	1.00148	1.0011	—	1.00157
β_2	$F = 250$	0.9974	0.9951	0.9953	0.997*
	$F = 500$	0.9974	0.9951	—	0.997

F is in MHz, $\phi_0 = 45°$.
*Frequency associated with these results in [7] is not mentioned. We assume (based on our calculations) that β_2 is almost independent of frequency in the above frequency region.

TABLE II
CONVERGENCE OF β_1, β_2 FOR THE EXPANSION OF EQUATION (11) WITH $\phi_0 = 45°$, $F = 500$, AND $n = 140$

Number of terms in equation (11)	β_1	β_2
2,2	1.00119	0.9947
4,4	1.00119	0.9949
6,6	1.00119	0.99508
8,8	1.00119	0.99510

Such very fast convergence suggests that the field slot as given by (11) is probably very close to the true field. In fact, as will be discussed later, the slot field coefficients are highly stable and are almost fixed as the number of expansion terms increases. Convergence of β_1 and β_2 on the assumption of (10) is not as good. Fig. 2 gives some examples of this fact. It is believed that the results of the development based on (11) are more accurate since the field solution of E_z approaches zero at the edge, as expected. Moreover, the results compare well with the analytical solution of Hurd [7] in addition to the high stability of the solution.

The results indeed show that the bifilar mode is faster than the speed of light in the dielectric filling medium and are in good agreement with the findings of [6] as well as with one of the developments presented in the work of Delogne and Laloux [7].

Field Solution

The field configuration on the slot as well as around the slotted cylinder is highly important for the proper coupling of the transreceivers to the structure. We present the slot

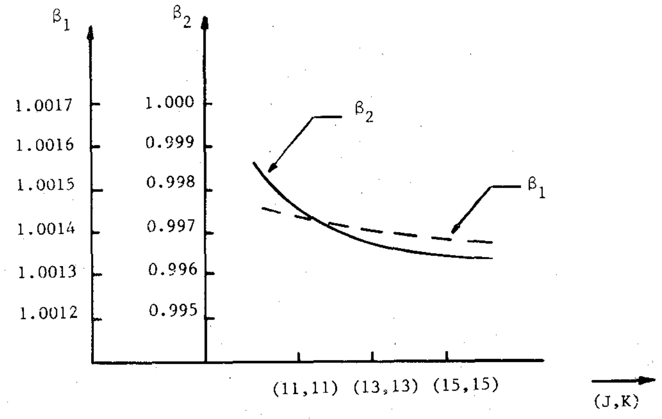

Fig. 2. Convergence of β_1, β_2 with the number of expansion terms of (10). $F = 500$ MHz, $\phi_0 = 45°$, $N = 140$.

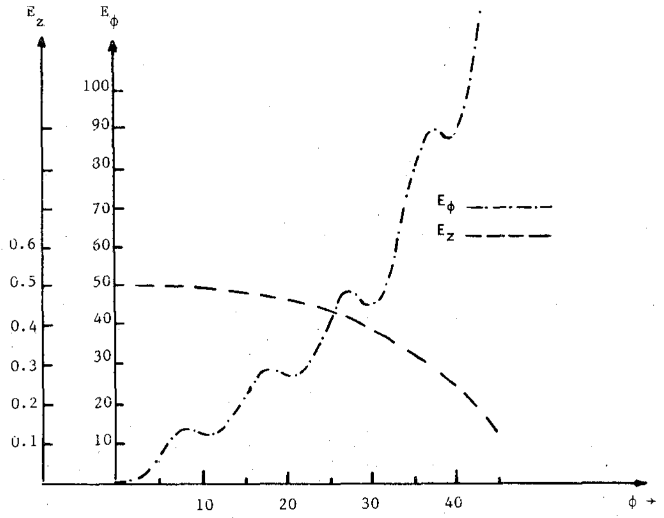

Fig. 3. Slot field configuration for the slot width $|\phi_0| = 45°$. $F =$ 500 MHz, the first coefficient of the E_ϕ expansion is taken equal to 100.

fields as obtained from set (11) since they seem to be stable and their fluctuation is slow as the number of expansion terms increases This fluctuation is expected and is a characteristic of the Ritz optimization technique [16]. Figs. 3 and 4 give an example of the field variation on the slot for two different slot widths. The results show that E_z is very small and that the slot electric field is essentially azimuthal. Further, it is noticed that the value of E_z approaches zero at the slot edge. This fact, as pointed out earlier, adds confidence to the obtained solution. Of course the field solution everywhere is now readily obtainable through (1)–(4) and (11).

One point of interest is the effect of E_z on the propagation constant α. It is noticed that the resulting value of E_z is very small compared to E_ϕ. This suggests that the solution of α should be insensitive to the choice of the basis function for E_z. In fact even if one sets $E_z = 0$ in (5), which leaves only the first term of the summation (6), the results of α (for the above parameters) show very little

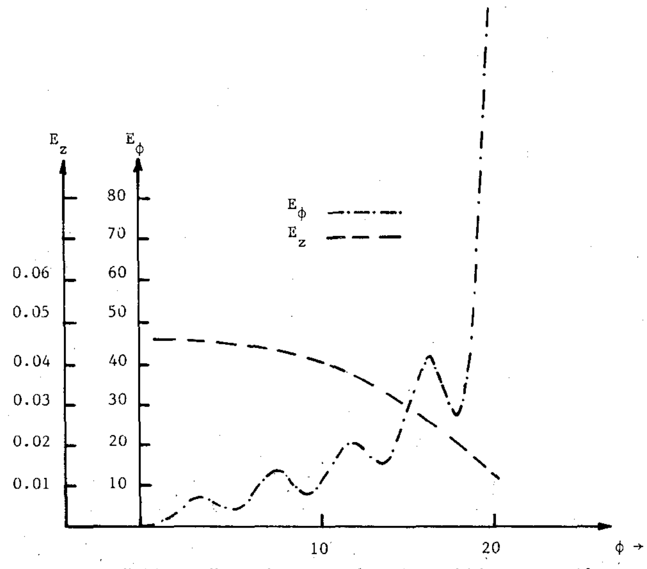

Fig. 4. Slot field configuration for the slot width $|\phi_0| = 20°$. $F =$ 250 MHz. The first coefficient of the E_ϕ expansion is taken equal to 100.

change from their original values. However, if such simplification is introduced, the ratio of wavelength to slot dimension must be taken into account. Similar simplification applied to microstrip line may be found in [11].

IV. Conclusion

A field solution and eigenmodes of the monofilar and bifilar modes of the slotted coaxial cable are presented. A variational technique is used to optimize the field distribution. It is found that a proper slot field would be one which has one singular term and an expansion of a complete set. Further, the results seem to resolve some conflicting concepts about the coaxial mode propagation constant where it is shown that this mode is faster than the corresponding speed of light in the dielectric media. Such results are important for the proper excitation as well as the correct termination of the slotted coaxial cable.

References

[1] D. J. Martin, "Radio communication in mines and tunnels," *Electron. Lett.*, no 6, pp. 563–564, 1970.

[2] J. C. Beal, J. Josiak, S. Mohamoud, and V. Rawat, "Continuous-access guided communication (CAGC) for ground transportation systems," *Proc. IEEE*, vol. 61, pp. 562–568, 1973.

[3] P. Delongne and L. Deryck, "Underground use of a coaxial cable with leaky sections," *IEEE Trans. Antennas Propagat.*, vol. AP-28, pp. 875–883, Nov. 1980.

[4] P. Degauque, B. Demoulin, J. Fontaine, and R. Gabillard, "Theory and experiment of mobile radio communication in tunnels by means of leaky braided coaxial cable," *Radio Sci.*, vol. 11, no. 4, pp. 305–314, Apr. 1976.

[5] P. Delogne, "Basic mechanism of tunnel propagation," *Radio Sci.*, vol 11, no. 4, pp. 259–330, Apr. 1976.

[6] R. Hurd, "The modes of an axially slotted coaxial waveguide," *Radio Sci.*, vol. 14, no. 5, pp. 741–751, Sept.–Oct. 1979.

[7] P. Delogne and A. Laloux, "Theory of slotted coaxial cable," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 1102–1107, Oct. 1980.

[8] P. Delogne, "Single mode excitation of a leaky feeder," *Rev. HF Electron Telecommun.*, vol. 13, no. 5, pp. 127–134, 1985.

[9] H. Kaden, "Open circularly cylindrical metal strip as shielded and return circuit of a single wire line," *Siemens Forsch-U. Entwickl.*, vol. 7, no. 2, pp. 82–90, 1978.
[10] E. E. Hassan, "Field solution, polarization and eigenmodes of shielded microstrip transmission line," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 845–852, Aug. 1986.
[11] E. E. Hassan, "Simple solution of dispersion characteristics of shielded strip line," *Proc. Inst. Elec. Eng.*, vol. 134, pt. H, no. 6, pp. 566–568, Dec. 1987.
[12] V. H. Rumey, "Travelling wave slot antenna," *J. Appl. Phys.*, vol. 24, no. 11, pp. 1358–1365, Nov. 1953.
[13] C. H. Walter, *Travelling Wave Antennas*. New York: McGraw-Hill, 1965, ch. 4.
[14] R. Harrington, "Propagation along a slotted cylinder," *J. Appl. Phys.*, vol. 24, pp. 1366–1371, Nov. 1953.
[15] P. Delogne, "Theory of the slotted coaxial cable," presented at URSI Conf., Helsinki, Aug. 2–8, 1978.
[16] M. Forray, *Variational Calculus in Science and Engineering*. New York: McGraw-Hill, 1968, ch. 7.

Essam E. Hassan was born in Alexandria, Egypt, in 1947. He received the B.Sc. degree in electrical engineering (with honors) from Alexandria University in 1970 and the M.Sc. and Ph.D. degrees in electrical engineering from the University of Manitoba, Winnipeg, Canada, in 1974 and 1978, respectively.

From 1977 to 1979, he was a Senior Research Engineer with Northern Telecom Canada in the digital switching division, where he was responsible for the design and testing of digital networks. In 1979, he joined the Department of Electrical Engineering at the University of Petroleum and Minerals, Dhahran, Saudi Arabia, where he is now an Associate Professor of Electrical Engineering. Since then he has been active in the fields of communication, microwave communication, and electromagnetic field theory.

Dr. Hassan is an author or coauthor of several technical papers on electromagnetics and communications as well as one book on communication theory.